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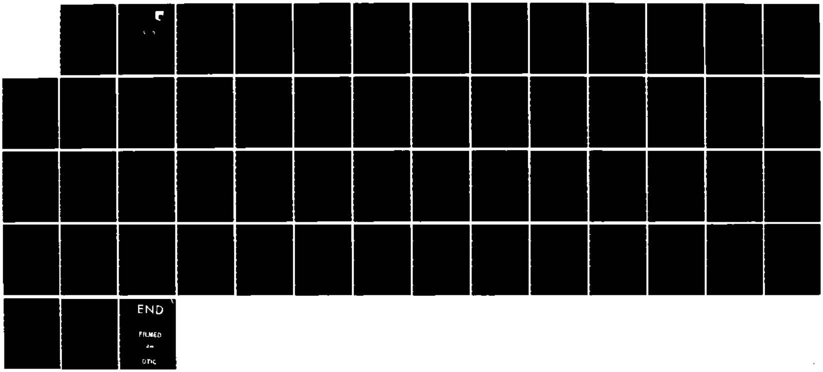

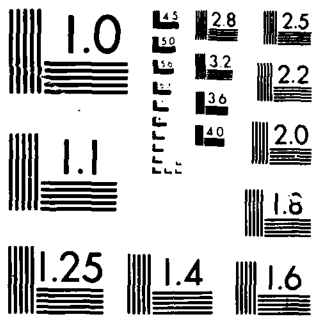
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TWO COMPLIANCE EXPRESSIONS FOR ARBITRARY LOCATIONS ACROSS A CRACK IN A CT SPECIMEN

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Two empirical equations were developed that express displacement across the crack in a compact type (CT) specimen as a function of crack length and distance behind the crack tip. The coefficients in the equations were generally determined by least square error fit to crack surface displacements obtained from finite element analysis. In one expression, single types of functions which depend on either crack length or distance from the crack tip were combined to form an equation that is similar to a series expansion in two separate variables. The second equation was developed from a combination of the far field behavior of the displacements and a near field fit which incorporates the near tip analytical displacement behavior. Both equations have application to a broad range of values of crack length and distance from the crack tip. Either equation can be used in conjunction with measurements of displacement across the crack surface to indirectly determine the crack length. [illegible]

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FOREWORD

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SECTION 1

INTRODUCTION

In the investigation of crack growth behavior using an interferometric displacement gage (IDG) [1]+, an empirical relationship between crack length and measured displacement under a load provides the basis for a nonvisual method of determining crack length. In theory, the displacement is assumed to be a homogeneous linear function of load. Hence, a single displacement measurement would be sufficient to define a load-displacement (P-δ) curve. However, in the laboratory, only a portion of the load-displacement curve is linear.

The linear portion is characterized by the compliance, C, which is the inverse of the slope, shown in Figure 1. This figure shows a typical load-displacement curve and a graphical definition of the compliance. Instead of using compliance which has units of length per force, a nondimensional quantity, EBC, will be used where E is an elastic modulus and B is the thickness of the specimen. The relationship between EBC and crack length developed in this report can be applied to specimens having a compact type (CT) configuration and to materials having an elastic response. In the following discussion, expressions are developed for the nondimensional compliance or, equivalently, the displacement between the crack surfaces as a function of distance from the crack tip and crack length.

+Numbers in Brackets Refer to References

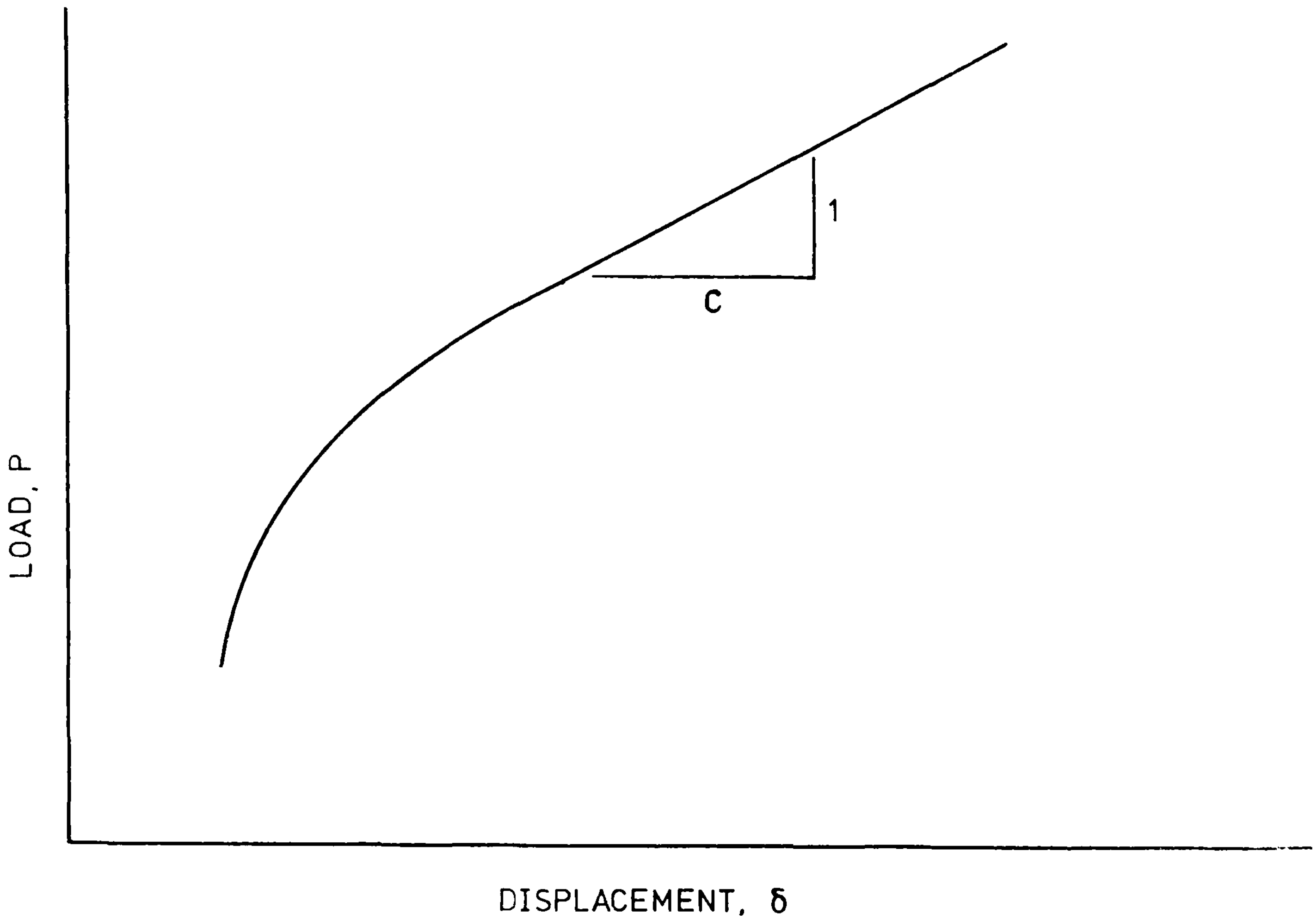

Figure 1. Illustration of Compliance Determination on Linear Portion of Load-Displacement Curve.

SECTION 2

GENERAL EXPRESSIONS FOR REPRESENTING DISPLACEMENTS

Using a general asymptotic form for crack surface displacement in [2], the nondimensional compliance can be written as:

$$EBC = A_0\sqrt{r} + A_1 r + A_2 r^{3/2} + \ldots \tag{1}$$

where r is the distance from the crack tip measured along the crack surface and A_0, A_1, ... are unknown coefficients. The number of coefficients could be terminated arbitrarily to produce any suitable degree of fit. Also, certain coefficients could be chosen to have specific values. From theoretical considerations, as $r \to 0$, the first coefficient can be related to the stress intensity factor, K,

$$A_0 = \frac{8B}{P\sqrt{2\pi}} K, \tag{2}$$

where B and P are the thickness and applied load, respectively,

$$K \equiv \lim_{x \to 0} \sqrt{2\pi x}\ \sigma_{yy}\ (x,0),$$

and x and σ_{yy} are the distance and normal stress, respectively, ahead of the crack tip.

Squaring (1) yields another form of a displacement expression

$$(EBC)^2 = A_0^2 r + A_1' r^{3/2} + A_2' r^2 + \ldots \tag{3}$$

where A_0, as in (1), and A_1', A_2', ... are unknown coefficients. Again, the expression can be terminated at any degree or some coefficients can be defined a priori.

Two representations will be developed. One is based on a truncation of the series expression. The other is based on a modification of a far field representation of the displacements. The truncated series representation is explored in the next section.

SECTION 3

EVALUATION OF COEFFICIENTS FOR TRUNCATED SERIES

In the development of an equation for the displacement, now expressed as EBC, for a wide range of values of crack length and distance from the tip, Orange's equation [3] which was developed for a bend type specimen was examined. Orange developed an equation by approximating the shape of the crack surface with a conic section. The equation has the form

$$\left(\frac{\delta}{\delta_0}\right)^2 = \frac{2}{2+m}\left(\frac{r}{a}\right) + \frac{m}{2+m}\left(\frac{r}{a}\right)^2, \tag{4}$$

where a is the crack length, δ_0 is the displacement of the crack surface at r = a under a given load, and m is the conic section coefficient. Note that only two nonzero coefficients in (3) are used to model (4). From analytical results for bend and single edge notch (SEN) specimens, the conic section coefficient can be expressed as a function of crack length ratio

$$m = -0.3 + 15\ (a/W)^n, \tag{5}$$

where n is 2.3 for bending or 3.3 for SEN, and W is the width of the specimen. However, extending an expression similar to (5) for CT specimens did not provide reasonable results for the displacement or, equivalently, the compliance for a wide range of values of a and r.

In the following paragraphs, the coefficients for various expressions of compliance will be evaluated from a least square error fit to analytical data on crack opening displacements. A finite element program described in [4] was used to generate the analytical values of EBC. The results are tabulated in Table 1 for various values of crack lengths, a/W, and r. Coordinates and notation for a CT specimen are shown in Figure 2.

3.1 TWO COEFFICIENT FIT FOR $(EBC)^2$

Equation (4) is a specific expression which incorporates only two unknowns, δ_0 and m, of the more general expression in (3). Even though m was the only unknown determined by an empirical fit, both δ_0 and m could be determined by this means. Using only the first two coefficients of integer powers in r from (3) as unknowns, compliance could be written as

$$(EBC)^2 = \alpha r + \beta r^2, \tag{6}$$

where α and β are unknown coefficients. Values of the coefficients, α and β, determined from the fit of the analytical data for various ratios of a/W are given in Table 2.

To examine the validity of the values of the coefficients, a comparison between (6) and (3) was made for $r \to 0$. Using (2), one obtains

$$\alpha = A_0^2 = [\frac{8}{\sqrt{2\pi}} \frac{B}{P} K]^2. \tag{7}$$

TABLE 1

ANALYTIC VALUES OF EBC FOR A CT SPECIMEN (W = 1.575 in)

a/W=0.25		a/W=0.30		a/W=0.35		a/W=0.40	
r (in)	EBC (-)	r (in)	EBC (-)	r (in)	EBC (-)	r (in)	EBC (-)
.0000E00	.00000E0	.0000E00	.00000E0	.0000E00	.00000E0	.0000E00	.00000E0
.1230E-2	.42967E0	.1480E-2	.54160E0	.1720E-2	.66471E0	.1970E-2	.81207E0
.4930E-2	.85801E0	.3910E-2	.10822E1	.6890E-2	.13327E1	.7880E-2	.16260E1
.6405E-2	.98272E0	.7685E-2	.12388E1	.8955E-2	.15242E1	.1024E-1	.18598E1
.7880E-2	.10998E1	.9460E-2	.13846E1	.1102E-1	.17014E1	.1260E-1	.20747E1
.1379E-1	.14352E1	.1656E-1	.18153E1	.1929E-1	.22387E1	.2205E-1	.27392E1
.1970E-1	.17354E1	.2365E-1	.21964E1	.2755E-1	.27089E1	.3150E-1	.33144E1
.2585E-1	.19978E1	.3449E-1	.26807E1	.4302E-1	.34397E1	.5166E-1	.43339E1
.3199E-1	.22295E1	.4532E-1	.30896E1	.5848E-1	.40471E1	.7181E-1	.51799E1
.4433E-1	.27200E1	.7060E-1	.39331E1	.9457E-1	.52835E1	.1188E00	.68995E1
.6066E-1	.31298E1	.9588E-1	.46439E1	.1307E00	.63425E1	.1659E00	.83988E1
.8933E-1	.38590E1	.1464E00	.59153E1	.2028E00	.82736E1	.2599E00	.11184E2
.1180E00	.44631E1	.1970E00	.70047E1	.2750E00	.99799E1	.3540E00	.13715E2

a/W=0.50		a/W=0.60		a/W=0.70		a/W=0.80	
r (in)	EBC (-)	r (in)	EBC (-)	r (in)	EBC (-)	r (in)	EBC (-)
.0000E00	.00000E0	.0000E00	.00000E0	.0000E00	.00000E0	.0000E00	.00000E0
.2460E-2	.12065E1	.2950E-2	.18737E1	.3450E-2	.32210E1	.3940E-2	.65825E1
[illegible]	.24251E1	.1181E-1	.37763E1	.1379E-1	.65079E1	.1575E-1	.13408E2
[illegible]	.27728E1	.1536E-1	.43189E1	.1793E-1	.74517E1	.2048E-1	.15405E2
[illegible]	.30914E1	.1890E-1	.48155E1	.2206E-1	.83164E1	.2520E-1	.17246E2
[illegible]	.41089E1	.3308E-1	.64461E1	.3861E-1	.11237E2	.4410E-1	.23672E2
[illegible]	.49754E1	.4725E-1	.78250E1	.5515E-1	.13717E2	.6300E-1	.29248E2
[illegible]	.67805E1	.8611E-1	.11000E2	.1034E00	.19899E2	.1206E00	.44282E2
[illegible]	.82933E1	.1250E00	.13706E2	.1516E00	.25305E2	.1781E00	.57982E2
[illegible]	.11419E2	.2156E00	.19432E2	.2642E00	.37064E2	.3124E00	.88718E2
[illegible]	.14245E2	.3063E00	.24784E2	.3768E00	.48354E2	.4468E00	.11887E3
[illegible]	.19663E2	.4877E00	.35286E2	.6019E00	.70840E2	.7154E00	.17925E3
[illegible]	.24812E2	.6690E00	.45576E2	.8270E00	.93247E2	.9840E00	.23976E3

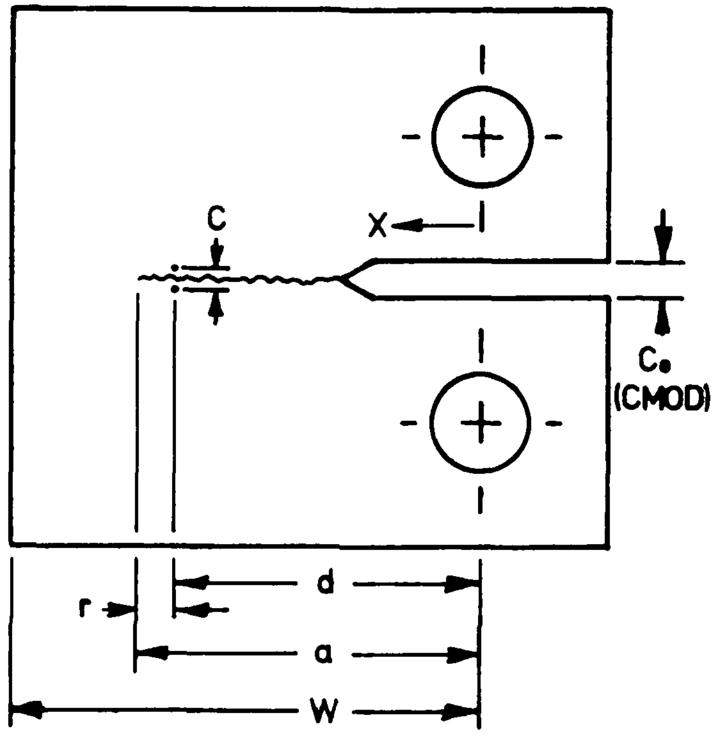

Figure 2. Coordinates and Notation to be Used for a CT Specimen.

TABLE 2

VALUES OF α, β, and $(K')^2$ VERSUS a/W

a/W	α	β	$(K')^2$
0.2500E+00	0.1525E+03	0.1437E+03	0.1568E+03
0.3000E+00	0.2017E+03	0.2435E+03	0.2043E+03
0.3500E+00	0.2598E+03	0.3744E+03	0.2642E+03
0.4000E+00	0.3343E+03	0.5582E+03	0.3426E+03
0.5000E+00	0.5724E+03	0.1230E+04	0.6034E+03
0.6000E+00	0.1102E+04	0.2990E+04	0.1206E+04
0.7000E+00	0.2678E+04	0.9158E+04	0.3004E+04
0.8000E+00	0.9615E+04	0.4954E+05	0.1098E+05

For a CT specimen, the stress intensity factor is given by

$$K = \frac{P}{B\sqrt{W}} f(\frac{a}{W}), \tag{8}$$

where from [5]

$$f(\frac{a}{W}) = \frac{2+a/W}{(1-\frac{a}{W})^{3/2}}(.886+4.64(\frac{a}{W})-13.32(\frac{a}{W})^2+14.72(\frac{a}{W})^3-5.6(\frac{a}{W})^4). \tag{9}$$

Defining a new quantity,

$$K' \equiv \frac{8}{\sqrt{2\pi W}} f(\frac{a}{W}), \tag{10}$$

and using (8), (7) can be restated as

$$\alpha = A_0^2 = (K')^2, \ r \to 0. \tag{11}$$

Since α was determined from a least square fit to numerical results, (11) may not be satisfied exactly. Values of α are compared with values of $(K')^2$ in Table 2. When a/W is small, α is in fairly good agreement with $(K')^2$, but as a/W increases, values of α deviate more from $(K')^2$.

Since the least square error fit using (6) follows the general trend of EBC, the compliance as $r \to 0$ may not be accurately described. The deviation in α from $(K')^2$ occurs because (6) is forced to fit larger values of EBC away from the crack tip as a/W increases. Thus, the compliance behavior further away from the crack tip becomes more dominant in the fit and the resulting equation does not characterize the near crack tip region.

Additional expressions for crack surface compliance derived from Equation (1) will be investigated. In the following equations, the near tip behavior is enforced analytically.

3.2 TWO COEFFICIENT FIT FOR $(EBC)^2$ BEYOND CRACK TIP VICINITY

From the theoretical considerations discussed earlier, the first coefficient in (3) is assumed to be given by (11). Also, it was decided that 'half-integer' powers should be considered. To maintain simplicity, the number of unknown coefficients should be kept small. To generalize (3), r was replaced with a nondimensional quantity, $\rho = r/W$, yielding:

$$(EBC)^2 = (K')^2 W\rho + c_0\rho^{3/2} + c_1\rho^2 \tag{12}$$

and

$$(EBC)^2 = (K')^2 W\rho + d_0\rho^2 + d_1\rho^3 \tag{13}$$

where c_0, c_1, d_0, and d_1 are the unknown coefficients. For a preliminary evaluation of the coefficients, the data for a/W = 0.8 was chosen because it appeared to be the most di.ficult to fit. The results of fitting (12) and

(13) are shown in Tables 3 and 4, respectively. The comparison of the root mean square deviation for the two fits indicates that the half-integer expression provides a slightly smaller error. Thus, Equation (12) will be investigated further.

3.3 TWO COEFFICIENT FIT FOR EBC BEYOND CRACK TIP VICINITY

Since a direct calculation of displacement would be more desirable than a squared expression, a truncated series from (1) using the result in (11) and expressed in terms of ρ was investigated,

$$EBC = \sqrt{W}\, K'\sqrt{\rho} + b_0\rho + b_1\rho^{3/2} \tag{14}$$

where b_0 and b_1 are unknown coefficients. The results from the fit using (14) are compared to the results using (12) in Table 5. Since the error using (14) was generally larger than the error using (12), the coefficients in (12) are determined for other values of a/W.

3.4 COEFFICIENTS AS FUNCTIONS OF a/W

From (12), values of c_0 and c_1 were determined for various values of a/W and are tabulated in Table 6. In an attempt to determine convenient expressions of c_0 and c_1 as a function of a/W, $-c_0$ and c_1 were first plotted against a/W on a log-linear scale. It appeared that $-c_0$ and c_1 could vary with respect to a/W in an exponential manner. Polynomial fits of the natural log of $-c_0$ and c_1 as a function of a/W were developed. Plots of the

TABLE 3

RESULTS FROM A LEAST SQUARE ERROR FIT
FOR a/W = 0.8 USING EQ. (12),

$$(EBC)^2=(K')^2W\rho+C_0\rho^{3/2}+C_1\rho^2$$

r/W	EBC Anal.	EBC Calc.	Percent Error
0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
0.2502E-02	0.6582E+01	0.6566E+01	-0.2533E+00
0.1000E-01	0.1341E+02	0.1335E+02	-0.4321E+00
0.1300E-01	0.1541E+02	0.1534E+02	-0.4198E+00
0.1600E-01	0.1725E+02	0.1716E+02	-0.5237E+00
0.2800E-01	0.2367E+02	0.2345E+02	-0.9505E+00
0.4000E-01	0.2925E+02	0.2894E+02	-0.1055E+01
0.7655E-01	0.4428E+02	0.4381E+02	-0.1061E+01
0.1131E+00	0.5798E+02	0.5761E+02	-0.6456E+00
0.1984E+00	0.8872E+02	0.8855E+02	-0.1879E+00
0.2837E+00	0.1189E+03	0.1189E+03	0.6660E-01
0.4542E+00	0.1793E+03	0.1794E+03	0.6460E-01
0.6248E+00	0.2398E+03	0.2397E+03	-0.1790E-01

TABLE 4

RESULTS FROM A LEAST SQUARE ERROR FIT FOR a/W = 0.8 USING EQ. (13),

$$(EBC)^2=(K')^2W\rho+d_0\rho^2+d_1\rho^3$$

r/W (-)	EBC Anal. (-)	EBC Calc. (-)	$\frac{\text{Calc.-Anal.}}{\text{Anal}}$ (%)
0.0000E+00	0.0000E+00	0.0000E+00	-
0.2502E-02	0.6582E+01	0.6629E+01	0.7125E+00
0.1000E-01	0.1341E+02	0.1357E+02	0.1182E+01
0.1300E-01	0.1540E+02	0.1561E+02	0.1320E+01
0.1600E-01	0.1725E+02	0.1747E+02	0.1299E+01
0.2800E-01	0.2367E+02	0.2391E+02	0.1009E+01
0.4000E-01	0.2925E+02	0.2951E+02	0.8845E+00
0.7655E-01	0.4428E+02	0.4452E+02	0.5402E+00
0.1131E+00	0.5798E+02	0.5830E+02	0.5553E+00
0.1984E+00	0.8872E+02	0.8897E+02	0.2865E+00
0.2837E+00	0.1189E+03	0.1190E+03	0.3970E+00
0.4542E+00	0.1793E+03	0.1791E+03	-.9090E-01
0.6248E+00	0.2398E+03	0.2398E+03	0.1670E-01

TABLE 5

COMPARISON OF RESULTS FROM LEAST SQUARE ERROR FITS FOR $a/W = 0.8$ USING EQ. (14), $EBC=K'\sqrt{W\rho}+b_0\rho+b_1\rho^{3/2}$, AND USING EQ. (12), $(EBC)^2=(K')^2W\rho+C_0\rho^{3/2}+C_1\rho^2$

$\rho=r/W$	EBC	Eq. (14),EBC		Eq. (12),EBC	
	Anal.	Calc.	$\frac{\text{Calc.-Anal.}}{\text{Anal.}}$	Calc.	$\frac{\text{Calc.-Anal.}}{\text{Anal.}}$
(-)	(-)	(-)	(%)	(-)	(%)
.000000	0.000000	0.000000	-	0.000000	-
.002502	6.582474	6.761556	2.7206	6.565800	-0.2533
.010000	13.407728	13.986365	4.3157	13.349794	-0.4321
.013000	15.405302	16.116470	4.6164	15.340630	-0.4198
.016000	17.246262	18.055210	4.6906	17.155947	-0.5237
.028000	23.672400	24.716805	4.4119	23.447420	-0.9504
.040000	29.247721	30.431873	4.0487	28.939106	-1.0552
.076549	44.281502	45.442307	2.6214	43.811771	-1.0608
.113098	57.981739	58.932022	1.6389	57.607437	-0.6456
.198375	88.718422	88.613770	-0.1180	88.551735	-0.1879
.283651	118.868523	117.847687	-0.8588	118.947716	0.0666
.454206	179.254944	177.780624	-0.8225	179.370804	0.0646
.624762	239.762405	240.848770	0.4531	239.719406	-0.0179

TABLE 6

VALUES FOR THE COEFFICIENTS RESULTING FROM EQ. (12),

$$(EBC)^2=(K')^2 W\rho+c_0\rho^{3/2}+c_1\rho^2$$

a/W	c_0	c_1
0.30	-24.16573	638.57910
0.35	-38.79956	981.62280
0.40	-69.99175	1474.47400
0.50	-231.55556	3311.06909
0.60	-697.99036	8111.06787
0.70	-1989.86206	25256.97852
0.80	-7595.10645	129157.88281

data and results of several polynomial fits to the data are shown in Figures 3 and 4. In Figure 3, the quadratic function for $\ln(-c_0)$ vs a/W appears to fit the data much better than the linear function. In Figure 4, the quadratic fit for $\ln(c_1)$ vs a/W at first seemed adequate, but after values of EBC were calculated for both the quadratic and cubic functions, the cubic expression was chosen due to the significantly improved error.

The resulting "best fit" expressions for c_0 and c_1 vs a/W are:

$$c_0 = \exp[\nu_0 + \nu_1(\frac{a}{W}) + \nu_2(\frac{a}{W})^2] \tag{15}$$

and

$$c_1 = \exp[\delta_0 + \delta_1(\frac{a}{W}) + \delta_2(\frac{a}{W})^2 + \delta_3(\frac{a}{W})^3] \tag{16}$$

where ν_0, ν_1, ν_2 and δ_0, δ_1, δ_2, δ_3 are constants.

3.5 EMPIRICAL EQUATION USING TRUNCATED SERIES

In summarizing the previous results, the following empirical equation which was based on a truncated series in $\rho^{1/2}$ was found to best represent the displacement along the crack surface as a function of crack length for a CT specimen:

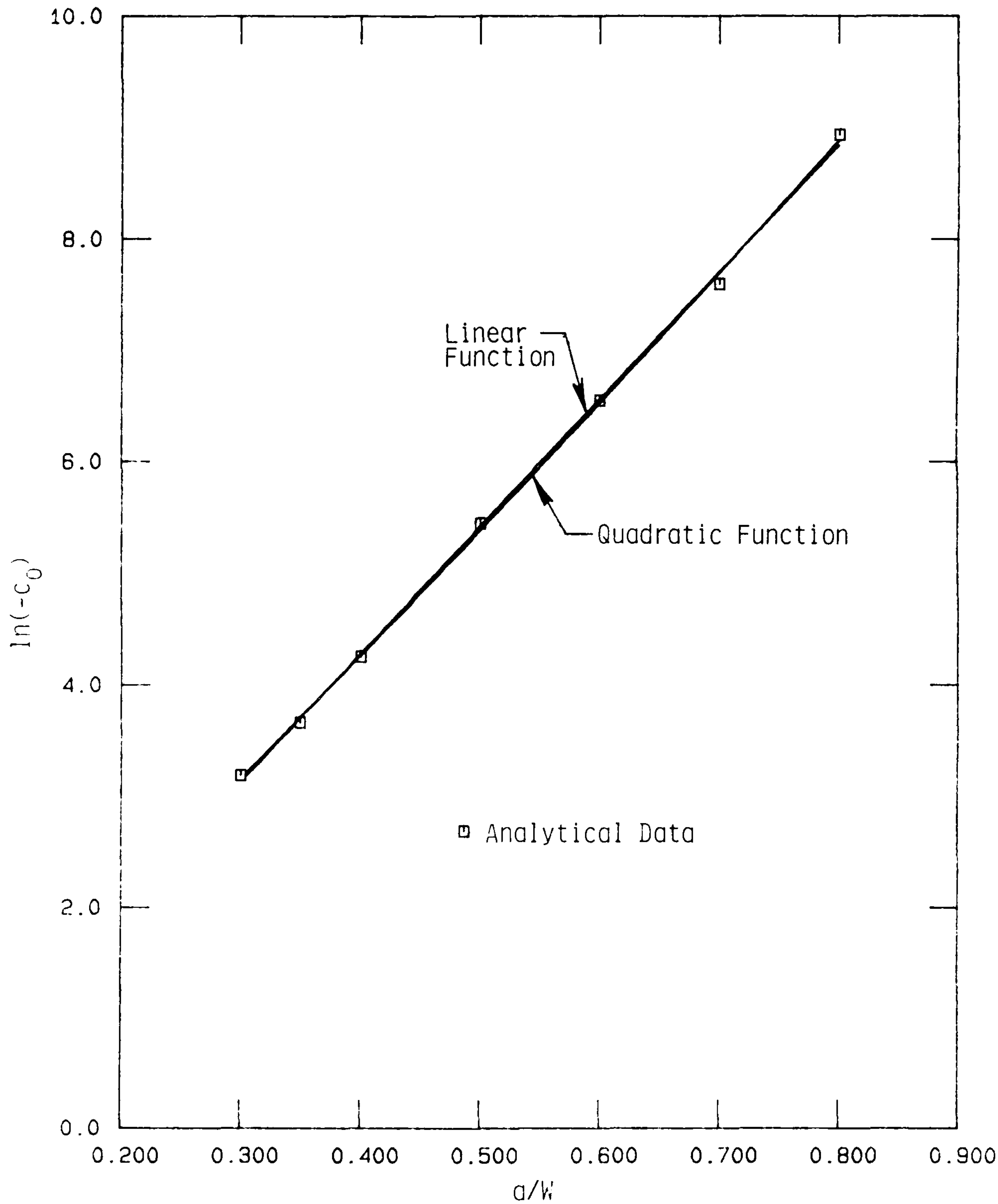

Figure 3. Comparison of Linear and Quadratic Fits of $\ln-(-c_0)$ to a/W.

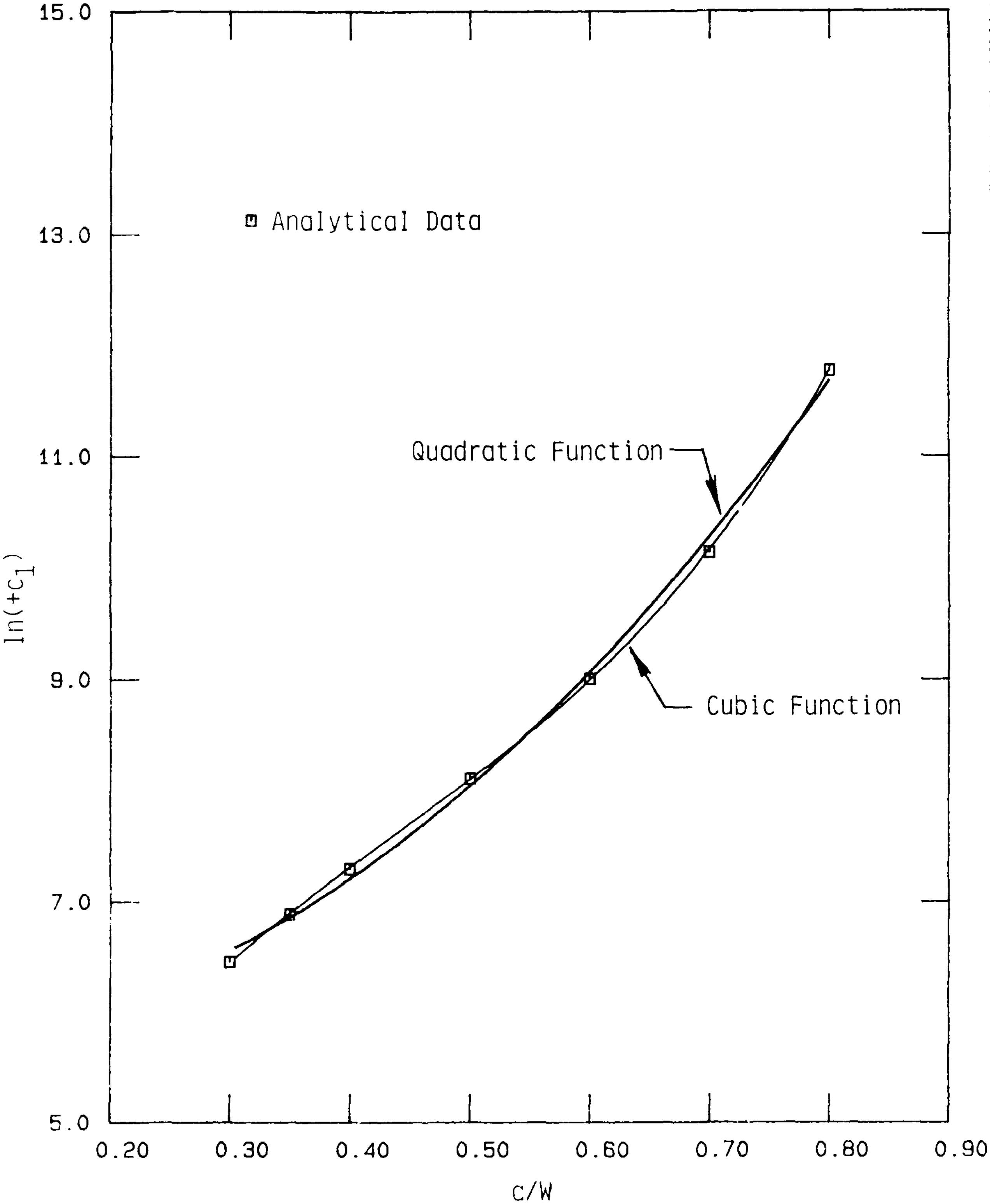

Figure 4. Comparison of Quadratic and Cubic Fits of $\ln(c_1)$.

$$EBC = [(\frac{8}{\sqrt{2\pi}}f(\frac{a}{W}))^2\rho+g(\frac{a}{W})\rho^{3/2}+h(\frac{a}{W})\rho^2]^{1/2} \tag{17}$$

where $\rho = r/W$, nondimensional distance behind the crack tip,

$$f(\frac{a}{W}) = \frac{(2+\frac{a}{W})}{(1-\frac{a}{W})^{3/2}}(.886+4.64(\frac{a}{W})-13.32(\frac{a}{W})^2+14.72(\frac{a}{W})^3-5.6(\frac{a}{W})^4), \tag{9}$$

$$g(\frac{a}{W}) = -\exp[\nu_0 + \nu_1(\frac{a}{W}) + \nu_2(\frac{a}{W})^2], \tag{18}$$

$$h(\frac{a}{W}) = \exp[\delta_0 + \delta_1(\frac{a}{W}) + \delta_2(\frac{a}{W}) + \delta_3(\frac{a}{W})^3], \tag{19}$$

$\nu_0 = 0.0285773$, $\delta_0 = 1.79452$,

$\nu_1 = 10.0701$, $\delta_1 = 24.1346$,

$\nu_2 = 1.25919$, $\delta_2 = -37.1842$,

$\delta_3 = 28.2394$.

A comparison of the desired values of EBC and the calculated results using Equation (17) is shown in Table 7. From the results in Table 7, the error is no longer greater than ± 3% over the ranges $0 \leq \rho \leq a/W - 0.175$ and $0.25 \leq a/W \leq 0.80$.

TABLE 7

RESULTS FROM THE TRUNCATED SERIES EXPRESSION, EQ. (17), TO CALCULATE EBC

a/W = 0.25

ρ=r/W (-)	EBC Anal. (-)	EBC Calc. (-)	(Calc.-Anal.)/Anal. (%)
.000000	0.000000	0.000000	-
.000781	0.429670	0.432968	0.7676
.003130	0.858010	0.867738	1.1338
.004067	0.982720	0.989578	0.6979
.005003	1.099800	1.098220	-0.1437
.008756	1.435200	1.456174	1.4614
.012508	1.735400	1.744700	0.5359
.016413	1.997800	2.003759	0.2983
.020311	2.229500	2.234934	0.2437
.029416	2.720000	2.706385	-0.5005
.038514	3.129800	3.116203	-0.4344
.056717	3.859000	3.829089	-0.7751
.074921	4.463100	4.455533	-0.1695

a/W = 0.30

ρ=r/W (-)	EBC Anal. (-)	EBC Calc. (-)	(Calc.-Anal.)/Anal. (%)
.000000	0.000000	0.000000	-
.000940	0.541600	0.542119	0.0959
.003752	1.082200	1.085152	0.2728
.004879	1.238800	1.238431	-0.0298
.006006	1.384600	1.375191	-0.6795
.010514	1.815300	1.826007	0.5898
.015016	2.196400	2.190332	-0.2763
.021898	2.680700	2.660598	-0.7499
.028775	3.089600	3.067915	-0.7019
.044825	3.933100	3.882429	-1.2883
.060876	4.643900	4.586820	-1.2291
.092952	5.915300	5.821053	-1.5933
.125079	7.004700	6.927941	-1.0958

TABLE 7 (Continued)

RESULTS FROM THE TRUNCATED SERIES EXPRESSION,
EQ. (17), TO CALCULATE EBC

a/W = 0.35

ρ=r/W (-)	EBC Anal. (-)	EBC Calc. (-)	(Calc.-Anal.)/Anal. (%)
.000000	0.000000	0.000000	-
.001092	0.664710	0.664469	-0.0363
.004375	1.332700	1.333024	0.0243
.005686	1.524200	1.521432	-0.1816
.006997	1.701400	1.689752	-0.6846
.012248	2.238700	2.246811	0.3623
.017492	2.708900	2.699108	-0.3615
.027314	3.439700	3.406576	-0.9630
.037130	4.047100	4.011774	-0.8729
.060044	5.283500	5.221133	-1.1804
.082984	6.342500	6.277997	-1.0170
.128762	8.273600	8.161949	-1.3495
.174603	9.979900	9.891178	-0.8890

a/W = 0.40

ρ=r/W (-)	EBC Anal. (-)	EBC Calc. (-)	(Calc.-Anal.)/Anal. (%)
.000000	0.000000	0.000000	.0[illegible]
.001251	0.812070	0.809429	-0.3253
.005003	1.626000	1.623625	-0.1461
.006502	1.859800	1.853545	-0.3363
.008000	2.074700	2.059201	-0.7471
.014000	2.739200	2.741730	0.0924
.020000	3.314400	3.299209	-0.4583
.032800	4.333900	4.287704	-1.0659
.045594	5.179900	5.130266	-0.9582
.075429	6.899500	6.824113	-1.0926
.105333	8.398800	8.327766	-0.8458
.165016	11.184000	11.061826	-1.0924
.224762	13.715000	13.623198	-0.6694

TABLE 7 (Continued)

RESULTS FROM THE TRUNCATED SERIES EXPRESSION, EQ. (17), TO CALCULATE EBC

a/W = 0.50

ρ=r/W (-)	EBC Anal. (-)	EBC Calc. (-)	$\frac{\text{Calc.-Anal.}}{\text{Anal.}}$ (%)
.000000	0.000000	0.000000	-
.001562	1.206500	1.198247	-0.6840
.006254	2.425100	2.406589	-0.7633
.008133	2.772800	2.750034	-0.8210
.010006	3.091400	3.056865	-1.1171
.017511	4.108900	4.081822	-0.6590
.025016	4.975400	4.927234	-0.9681
.043771	6.780500	6.684945	-1.4093
.062521	8.293300	8.192843	-1.2113
.106286	11.419000	11.296827	-1.0699
.150032	14.245000	14.130841	-0.8014
.237587	19.663000	19.475500	-0.9536
.325079	24.812000	24.626057	-0.7494

a/W = 0.60

ρ=r/W (-)	EBC Anal. (-)	EBC Calc. (-)	$\frac{\text{Calc.-Anal.}}{\text{Anal.}}$ (%)
.000000	0.000000	0.000000	-
.001873	1.873700	1.850269	-1.2505
.007498	3.776300	3.717831	-1.5483
.009752	4.318900	4.251172	-1.5682
.012000	4.815500	4.729215	-1.7918
.021003	6.446100	6.336740	-1.6965
.030000	7.825000	7.677175	-1.8891
.054673	11.000000	10.768578	-2.1038
.079365	13.706000	13.470174	-1.7206
.136889	19.431999	19.178482	-1.3046
.194476	24.784000	24.553379	-0.9305
.309651	35.285999	34.946835	-0.9612
.424762	45.576000	45.160328	-0.9120

TABLE 7 (Concluded)

RESULTS FROM THE TRUNCATED SERIES EXPRESSION, EQ. (17), TO CALCULATE EBC

a/W = 0.70

ρ=r/W (-)	EBC Anal. (-)	EBC Calc. (-)	(Calc.-Anal.)/Anal. (%)
.000000	0.000000	0.000000	-
.002190	3.221000	3.154046	-2.0787
.008756	6.507900	6.351496	-2.4033
.011384	7.451700	7.272814	-2.4006
.014006	8.316400	8.103281	-2.5626
.024514	11.237000	10.931138	-2.7219
.035016	13.717000	13.333463	-2.7961
.065651	19.899000	19.366501	-2.6760
.096254	25.305000	24.783155	-2.0622
.167746	37.063999	36.634975	-1.1575
.239238	48.354000	48.083637	-0.5591
.382159	70.839996	70.633438	-0.2916
.525079	93.247002	93.067078	-0.1930

a/W = 0.80

ρ=r/W (-)	EBC Anal. (-)	EBC Calc. (-)	(Calc.-Anal.)/Anal. (%)
.000000	0.000000	0.000000	-
.002502	6.582500	6.468560	-1.7309
.010000	13.408000	13.162839	-1.8285
.013003	15.405000	15.132161	-1.7711
.016000	17.246000	16.925253	-1.8598
.028000	23.672001	23.154879	-2.1845
.040000	29.247999	28.601858	-2.2092
.076571	44.282001	43.392429	-2.0089
.113079	57.981998	57.112049	-1.5004
.198349	88.718002	87.953712	-0.8615
.283683	118.870003	118.289337	-0.4885
.454222	179.250000	178.560394	-0.3847
.624767	239.759995	238.760498	-0.4169

SECTION 4

EVALUATION OF A LINEAR EXPRESSION WITH CORRECTION TERM

An expression for the crack surface displacement as a function of crack length for a CT specimen has been developed by Saxena and Hudak [6]. The form of the expression is desirable because of its simplicity. In terms of compliance, the expression was

$$EBC = \frac{X_R/W - X/W}{X_R/W + 0.25}\, EBC_0 \tag{20}$$

where X and X_R are the distances from the load line to the location of displacement on the crack surface and to the axis of rotation*, respectively, and C_0 is the compliance at the crack mouth. In [6], values of X_R are tabulated for various values of a/W and C_0 is given by the following empirical function

$$EBC_0=\left(1+\frac{0.25}{a/W}\right)\left(\frac{1+a/W}{1-a/W}\right)^2[1.61369+12.6778(a/W)-14.2311(a/W)^2$$
$$-16.6102(a/W)^3+35.0499(a/W)^4-14.4943(a/W)^5]. \tag{21}$$

For a given a/W, (20) is linear in X or in r since X = a - r.

*The axis of rotation is defined in [6].

Essentially, (20) is a good representation of the crack surface displacements in the vicinity of the load line. A plot of the finite element data in Figure 5 for a relatively short crack, $a/W = 0.25$, in a CT specimen shows that the linear behavior is a good approximation to the compliance in the vicinity of the load line, i.e., $0.1 < \rho < 0.6$. However, note that the linear approximation does not pass through the crack tip where the compliance should be zero.

4.1 CORRECTION TERM

The addition of a correction term to the linear compliance expression would be necessary to obtain the near tip behavior. Thus, a representation for a general compliance expression, cf. Equation (1), could be written as

$$C/C_0 = a_0 + a_1 \rho - (\text{correction term}), \tag{22}$$

where a_0 and a_1 are functions of a/W and the correction term is a function of a/W and ρ.

Comparing the linear terms of (20) when $X = a - r$ and (22), the coefficients in (22) could be approximated by

$$a_0 \approx \frac{X_R/W - a/W}{X_R/W + 0.25} \text{ and } a_1 \approx \frac{1}{X_R/W + 0.25} . \tag{23}$$

Values of a_0 and a_1 which are calculated from (23) using the values of X_R which are given in [6] are listed in Table 8.

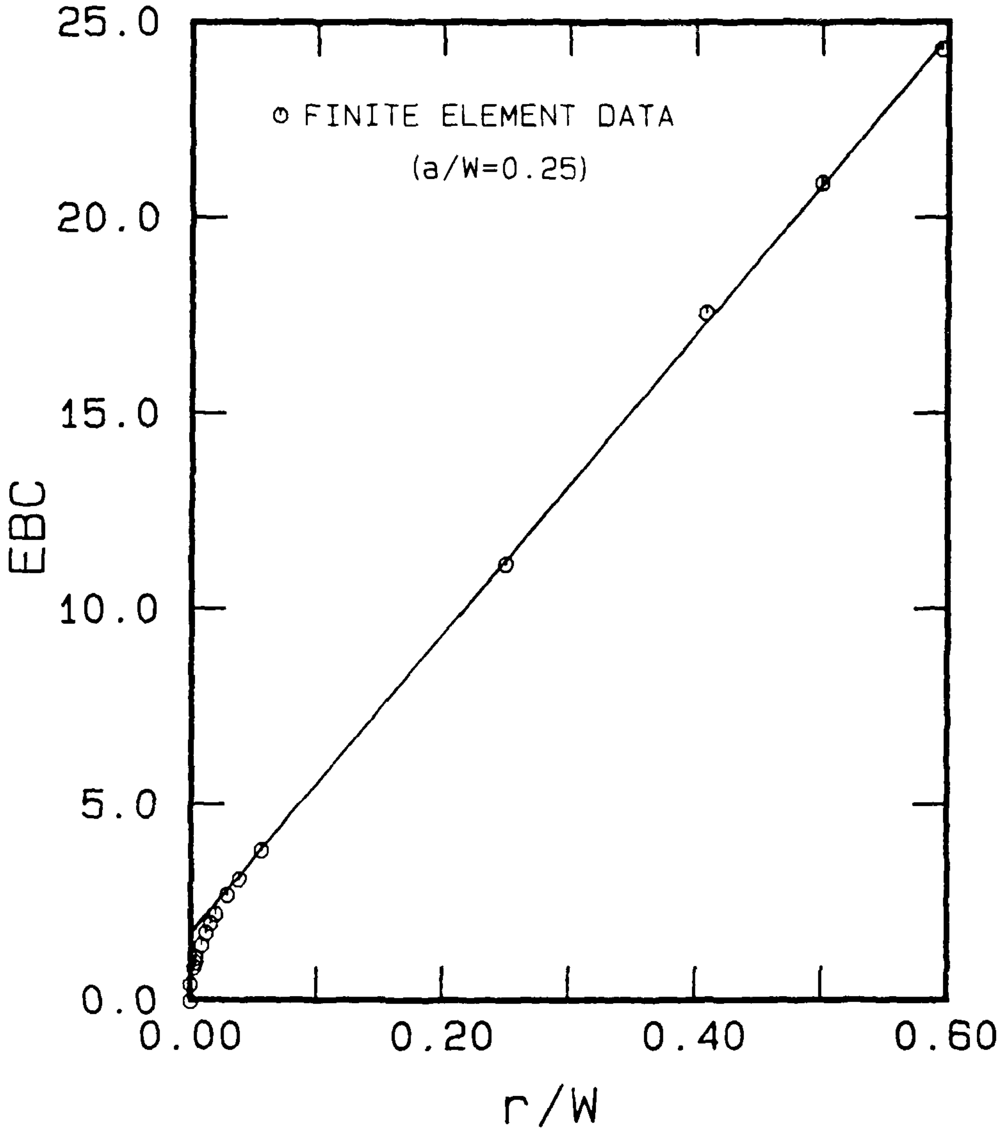

Figure 5. Finite Element (Analytical) Data for the Displacement Between the Crack Surfaces in a CT Specimen with a/W = 0.25.

TABLE 8

ESTIMATED VALUES OF THE COEFFICIENTS FOR THE LINEAR TERMS IN EQ. (22)

a/W	a_0	a_1
0.2500E+00	0.1380E+00	0.1720E+01
0.3000E+00	0.2040E+00	0.1590E+01
0.3500E+00	0.1070E+00	0.1490E+01
0.4000E+00	0.9190E-01	0.1400E+01
0.5000E+00	0.6320E-01	0.1250E+01
0.6000E+00	0.4330E-01	0.1130E+01
0.7000E+00	0.3710E-01	0.1014E+01
0.8000E+00	0.2960E-01	0.9240E+00

In developing the correction term, the theoretical behavior of the compliance as $\rho \to 0$ which is described in (2) and (1) will be imposed. From the result in (2) and using (8) and (10), the correction term in (22) must have the following behavior,

$$(\text{correction term}) - a_0 = -\frac{K'\sqrt{W}}{EBC_0}\sqrt{\rho},\quad \rho \to 0. \tag{24}$$

The correction term must also approach zero far from the crack tip so that the form of the Saxena-Hudak expression is recovered. Various selections for the functional form of the correction term are discussed later.

4.2 SOLUTION TECHNIQUE

The procedures which are used to determine the unknown quantities in (22) involve a fit to the known data and an application of a measure of the nearness of the fit. Since (22) is expected to be nonlinear with respect to the unknown coefficients, a direct solution is not generally possible. Two types of iterative techniques are considered to obtain a solution for the unknowns.

One technique is to select initial values for those unknown quantities which contribute the nonlinear terms in (22) and solve in a least square error sense the unknowns in the linear terms. Then, update the values of the coefficients in the nonlinear terms with the most recent solution and repeat the least square error solution. The iteration was terminated when the solution converges to within a small fixed percent of the preceding solution.

A second technique which is basically a trial and error method uses an initial guess for the solution of the unknowns and then the deviations and percent errors in the calculated compliances are evaluated. The initial guess is incremented and a new set of deviations are calculated. Successive increments are made to the values of the unknowns until the deviations are acceptable.

Two criteria are applied to the deviations. For one criterion, the root mean square deviation (RMSD) is examined for a minimum. For the second criterion, the distribution of the percent errors of the deviations is examined for uniform scatter about the desired values. Generally, these two criteria cannot be satisfied simultaneously. However, a solution for the unknowns is selected when the RMSD is near a minimum value in order that the maximum error in the scatter is reduced to a small value. A result based on this subjective technique can be obtained rather quickly if the initial guess is close to a minimum RMSD. An example of the RMSD using this procedure is given in Section 4.4.

4.3 FINITE ELEMENT RESULTS FOR COMPLIANCE

The unknowns in (22) are determined so that the calculated values of compliance from (22) agree with known values of compliance. Since Newman's results [7] have been used to develop various expressions for compliance or displacement in the vicinity of the load line for a CT specimen, those results are combined with the finite element results determined for this study for the intermediate distances between the load line and the crack tip. These combined values are shown in Table 9. It is noted that the finite element results from the analysis used for this report did not significantly differ from Newman's results where the two sets of results overlapped. The data listed in Table 9 are used to determine the coefficients in the subsequent expressions for compliance.

4.4 EXPRESSIONS FOR THE CORRECTION TERM

Various functional expressions for the correction term in (22) were evaluated. For all the expressions, the near crack tip behavior given in (24) is satisfied analytically. One of the expressions is

$$\frac{C}{C_0} = \frac{a_0}{n} + a_1\rho - \frac{a_0}{n} e^{-\frac{n}{a_0}\lambda\sqrt{\rho}} \tag{25}$$

where $\lambda = K'\sqrt{W}/(EBC_0)$ and n is an unknown constant which was selected to have integer values. The first solution technique, described in Section 4.2, was used to determine the coefficients, a_0 and a_1, with their initial values given in Table 8. Solutions were obtained but the values of a_0 and a_1 differed significantly from the initial values such that if values of ρ were larger than those used in region of fit, the solution was not realistic.

Another expression for the correction term in (22) yields

$$\frac{C}{C_0} = a_0 + a_1\rho - f(\sqrt{\rho})/g(\sqrt{\rho}), \tag{26}$$

TABLE 9

[illegible] VALUES OF EBC FOR A CT SPECIMEN (W=1.575 in)
(VALUES FROM REF. 7 INDICATED BY AN ASTERISK)

a/W=0.25		a/W=0.30		a/W=0.35		a/W=0.40	
r (in)	EBC (-)	r (in)	EBC (-)	r (in)	EBC (-)	r (in)	EBC (-)
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
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[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]*	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]*
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]*	[illegible]	[illegible]*	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]*	[illegible]	[illegible]*	[illegible]	[illegible]*	[illegible]	[illegible]*
				[illegible]	[illegible]*	[illegible]	[illegible]*

a/W=0.50		a/W=0.60		a/W=0.70		a/W=0.80	
r (in)	EBC (-)	r (in)	EBC (-)	r (in)	EBC (-)	r (in)	EBC (-)
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	.13790E-1	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	.43189E1	.17925E-1	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	.48115E1	.22060E-1	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	.33075E-1	.64461E1	.38605E-1	[illegible]	.44100E-1	[illegible]
[illegible]	[illegible]	[illegible]	.73350E1	.55150E-1	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	.11000E2	.10339E00	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	.15163E00	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	.26419E00	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	.37475E00	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]*	[illegible]	[illegible]*	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]*	[illegible]	[illegible]*
[illegible]	[illegible]*	[illegible]	[illegible]*	.11025E01	[illegible]*	[illegible]	[illegible]*
[illegible]	[illegible]*	[illegible]	[illegible]*	[illegible]	[illegible]*	[illegible]	[illegible]*

where f and g are polynomial functions in $\sqrt{\rho}$. Only polynomial equations involving no more than two coefficients in each function were investigated. Again, the first solution technique in Section 4.2 was used to determine the unknown coefficients in (26). Generally, the results from the least square error solution did not fit the intermediate compliance data very well. Also, the correction term did not approach zero rapidly enough as ρ increased.

Since the exponential behavior appeared to produce a better fit, various other expressions involving exponential behavior were tried. The following expression for the correction term produced a good fit

$$\text{(correction term)} = \frac{(1+a_2)a_0}{1+a_2\ \exp[\frac{1+a_2}{a_2 a_0}\lambda\sqrt{\rho}]} \tag{27}$$

where a_2 is also an unknown quantity. For any value of a_2, the near crack tip behavior in (24) is satisfied. To minimize the number of coefficients which are functions of a/W in (27), i.e., to maintain a simple expression for the compliance, a_2 is taken to be a constant.

Using the data for a/W = 0.5, several values of a_2 were chosen and the second solution technique in Section 4.2 was used to determine a_0 and a_1. The second technique was applied because the first technique did not yield acceptable results for the exponential expression in (27). The value of a_2 which produced a good result was approximately 2. The value, $a_2 = 2$, was used when solutions for a_0 and a_1 were obtained for other values of a/W.

As an example of the application of the second solution technique when (27) is used in (22), increments were subtracted and added to the initial values of a_0 and a_1 which were chosen from Table 8 for a given a/W value. Compliance values, percent errors and a RMSD were calculated for each combination of a_0 and a_1. An array of RMSD was created for the values of a_0 and the values of a_1 as illustrated in Table 10. The percent errors in calculated compliances were examined for the values of a_0 and a_1 in the vicinity of the minimum RMSD. If a set of percent errors was uniformly distributed within a ± 3% scatter band then the values of a_0 and a_1 for that set were selected as a solution.

If none of the scatter bands were within ± 3%, then smaller increments would be applied to the values of a_0 and a_1 at the smallest RMSD value to define more accurately the values of a_0 and a_1 for a minimum RMSD. Again, the percent errors would be examined for a uniform distribution of errors within a ± 3% scatter band. The procedure was repeated until a ± 3% scatter band was achieved or the smallest increments of change were 0.0005 for a_0 and 0.005 for a_1. Then, values of a_0 and a_1 were selected. The values of a_0 and a_1 which were chosen as solutions for each a/W are listed in Table 11.

Next, a_0 and a_1 were considered as functions of a/W. A least square error polynomial fit was used. Figures 6 and 7 show a_0 and a_1 plotted against a/W to illustrate the behavior of the functions. A second order polynomial proved satisfactory for a_1 and a third order polynomial was needed for a_0. The order of the polynomials was kept small as possible to preserve the simplicity and yet maintain the accuracy of the fit.

TABLE 10

EXAMPLE OF RMSD ARRAY USED TO EVALUATE a_0 AND a_1 FOR EQ. (27) SUBSTITUTED INTO (22) WHEN a/W = 0.4

a_1 \ a_0	.0800	.0825	.0850	.0875	.0900	.0925	.0950	.0975	.1000
1.300	0.9063	0.8567	0.8089	0.7631	0.7193	0.6777	0.6387	0.6025	0.5695
1.325	0.7131	0.6631	0.6154	0.5701	0.5277	0.4886	0.4533	0.4225	0.3970
1.350	0.5254	0.4754	0.4284	0.3852	0.3466	0.3140	0.2889	0.2730	0.2674
1.375	0.3524	0.3047	0.2626	0.2285	0.2057	0.1975	0.2050	0.2260	0.2566
1.400	0.2300	0.2005	0.1867	0.1912	0.2122	0.2449	0.2848	0.3289	0.3753
1.425	0.2491	0.2609	0.2851	0.3186	0.3582	0.4018	0.4480	0.4957	0.5444
1.450	0.3896	0.4201	0.4564	0.4969	0.5404	0.5861	0.6331	0.6811	0.7296
1.475	0.5675	0.6045	0.6448	0.6877	0.7324	0.7785	0.8255	0.8732	0.9214
1.500	0.7569	0.7968	0.8389	0.8828	0.9280	0.9742	1.0211	1.0685	1.1162

TABLE 11

FINAL VALUES OF THE COEFFICIENTS FOR THE LINEAR TYPE EQUATION WITH CORRECTION TERM

a/W	a_0	a_1
0.2500E+00	0.8800E-01	0.1820E+01
0.3000E+00	0.9320E-01	0.1637E+01
0.3500E+00	0.9450E-01	0.1485E+01
0.4000E+00	0.9250E-01	0.1375E+01
0.5000E+00	0.8250E-01	0.1205E+01
0.6000E+00	0.6900E-01	0.1090E+01
0.7000E+00	0.6100E-01	0.9750E+00
0.8000E+00	0.4230E-01	0.9085E+00

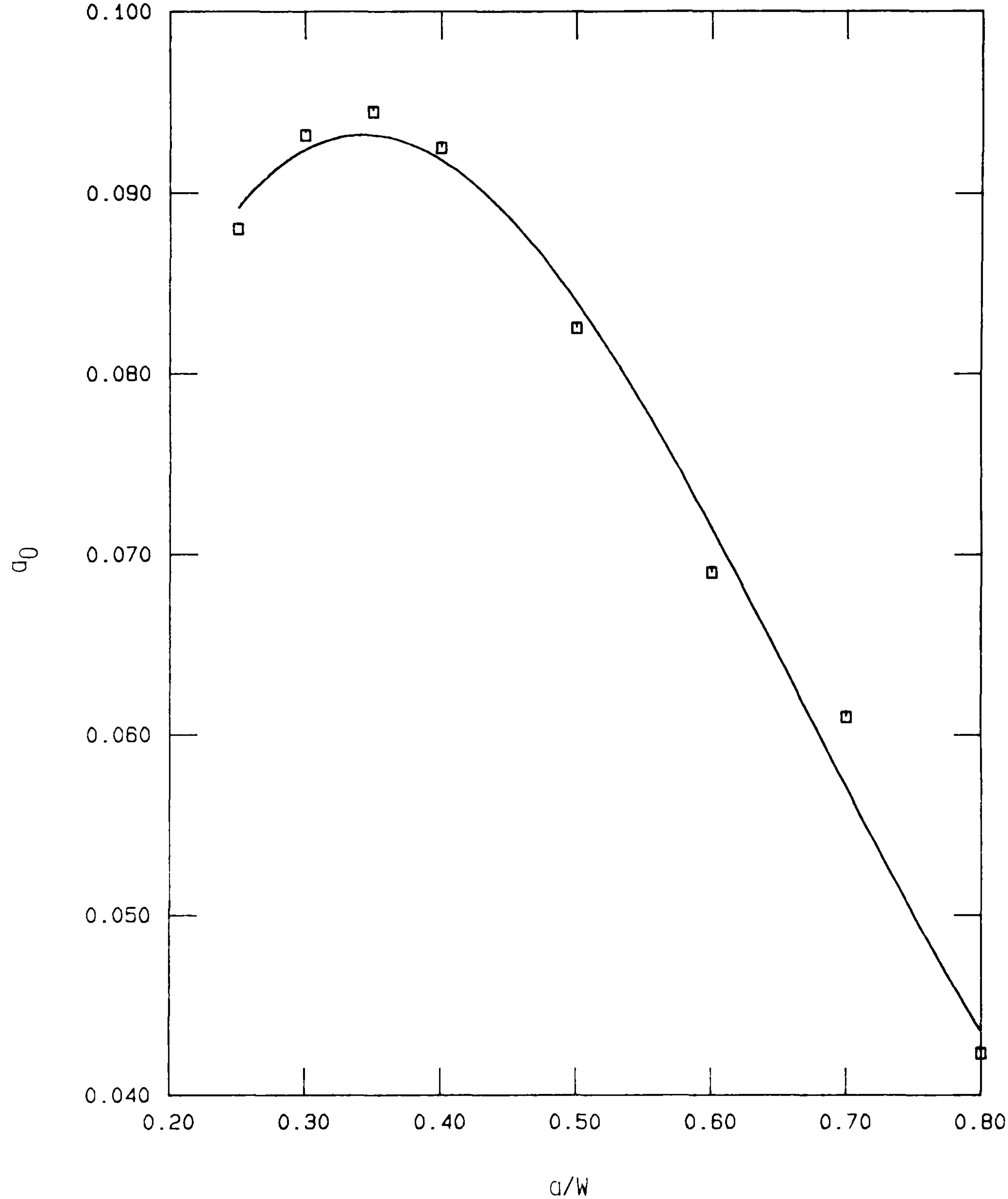

Figure 6. Values of a_0 Obtained for the Compliance Expression with the Correction Term and the Least Square Error Curve Fit.

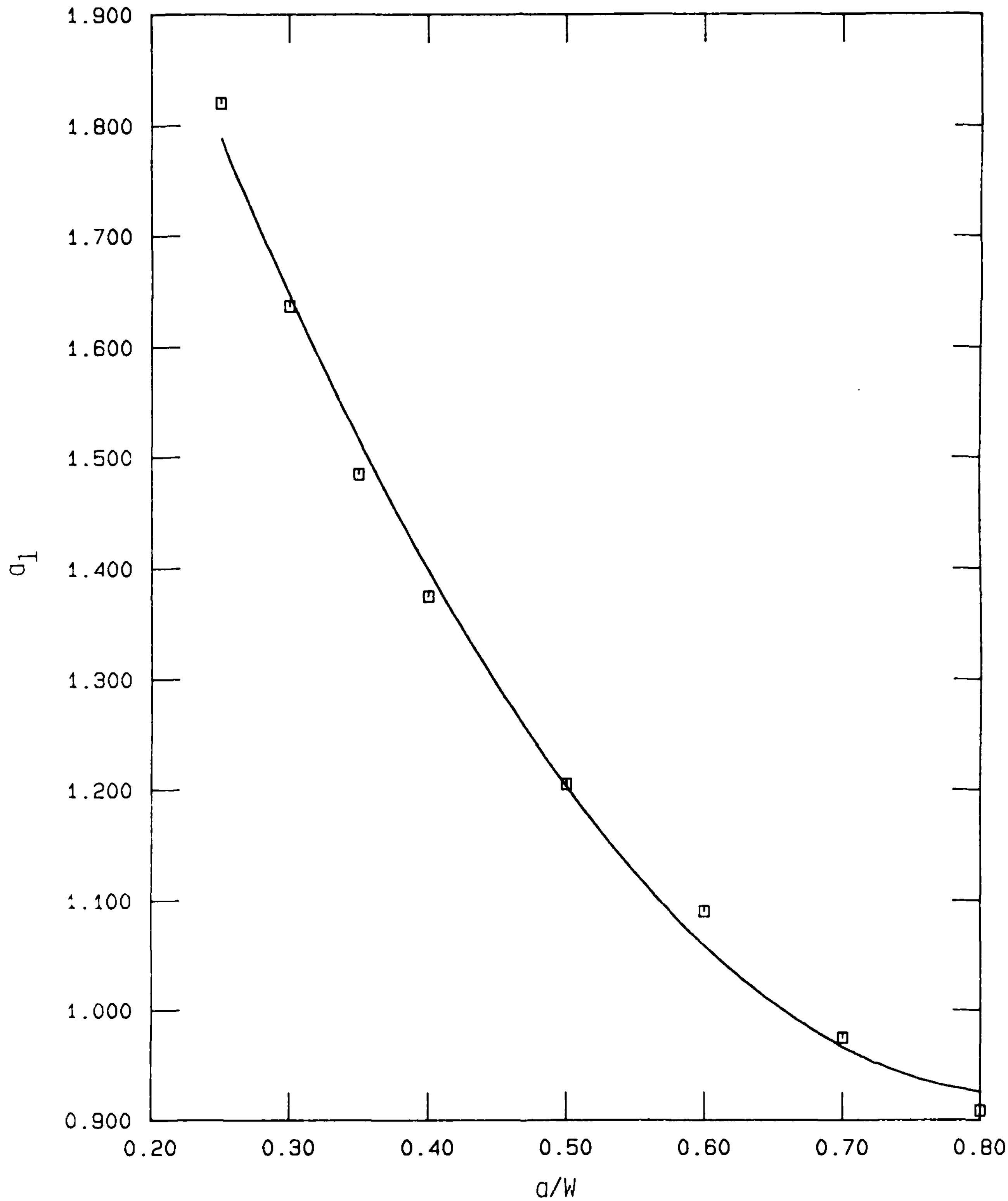

Figure 7. Values of a_1 Obtained for the Compliance Expression with the Correction Term and the Least Square Error Curve Fit.

4.5 EMPIRICAL EQUATION USING CORRECTION TERM

The relationship developed in this section has the final form:

$$\frac{C}{C_0} = a_0 + a_1\rho - \frac{3a_0}{1+2\cdot\exp[1.5\lambda\sqrt{\rho}/a_0]} \;, \tag{28}$$

where

C_0 is the compliance at the crack mouth opening, Eq. (21),

$\lambda = K'\sqrt{W}/(EBC_0)$,

$a_0 = .024 + .457(a/W) - .897(a/W)^2 + .445(a/W)^3$,

$a_1 = 2.696 - 4.279(a/W) + 2.582(a/W)^2$,

and

K' is given by (10) and (9).

A comparison of the desired values of EBC and the calculated results using (28) is shown in Table 12. From the results in Table 12, the error is no greater than ± 4% over the ranges $0 \leq \rho \leq a/W + 0.25$ and $0.25 \leq a/W \leq 0.80$. Note that (28) produces a slightly larger deviation from the desired values than (17) but (28) covers a wider range of distances from the crack tip than (17). Also, (28) is expected to be applicable for values of ρ beyond $a/W + 0.25$.

TABLE 12

RESULTS FROM THE LINEAR EXPRESSION
PLUS CORRECTION TERM, EQ. (28), TO CALCULATE EBC

a/W = 0.25

ρ=r/W (-)	EBC Anal. (-)	EBC Calc. (-)	(Calc.-Anal.)/Anal. (%)
0.0000	0.00000	0.00000	-
0.0015	0.41[illegible]	0.43957	1.39657
0.0031	0.85801	0.87557	2.04625
0.0041	0.98272	0.99573	1.3240[illegible]
0.0050	1.09980	1.10218	0.21598
0.0066	1.43520	1.44851	0.95944
0.0125	1.73540	1.72587	-0.54889
0.0164	1.99780	1.9742	-1.1779[illegible]
0.0203	2.22730	2.19765	-1.32731
0.0294	2.72000	2.66160	-2.14097
0.0385	3.12980	3.07988	-1.59485
0.0567	3.85900	3.85041	-0.22196
0.2500	11.18000	11.20418	0.21627
0.5000	20.91000	20.55361	-1.70726

RMSD ERROR = 0.1066

a/W = 0.30

ρ=r/W (-)	EBC Anal. (-)	EBC Calc. (-)	(Calc.-Anal.)/Anal. (%)
0.0000	0.00000	0.00000	-
[illegible]	0.54160	0.55223	1.9[illegible]
[illegible]	[illegible]	1.09958	[illegible]
[illegible]	[illegible]	1.25190	[illegible]
[illegible]	[illegible]	1.38697	[illegible]
[illegible]	[illegible]	1.8272[illegible]	[illegible]
[illegible]	[illegible]	[illegible]18041	-0.72804
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]03547	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]

RMSD ERROR = 0.0995

TABLE 12 (Continued)

RESULTS FROM THE LINEAR EXPRESSION
PLUS CORRECTION TERM, EQ. (28), TO CALCULATE EBC

a/W = 0.35

ρ=r/W (-)	EBC Anal. (-)	EBC Calc. (-)	(Calc.-Anal.)/Anal. (%)
[illegible]	[illegible]	0.00000	-
[illegible]	0.66971	0.67863	[illegible]
[illegible]	[illegible]33270	1.35654	1.78911
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]70140	1.71259	[illegible]
[illegible]	2.23670	2.26091	[illegible]
[illegible]	2.70890	2.70260	-0.23279
[illegible]	3.43970	3.39322	[illegible]
[illegible]	[illegible]04710	3.99212	-1.35260
[illegible]	[illegible]64000	4.54449	-2.05846
[illegible]	[illegible]	[illegible]23011	-1.01045
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	8.53548	3.16522
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	29.89000	[illegible]	[illegible]19167

RMSD ERROR = 0.1703

a/W = 0.40

ρ=r/W (-)	EBC Anal. (-)	EBC Calc. (-)	(Calc.-Anal.)/Anal. (%)
[illegible]	[illegible]	[illegible]	-
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]63847	[illegible]
[illegible]	[illegible]	[illegible]86987	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]

RMSD ERROR = 0.2237

TABLE 12 (Continued)

RESULTS FROM THE LINEAR EXPRESSION
PLUS CORRECTION TERM, EQ. (28), TO CALCULATE EBC

a/W = 0.50

ρ=r/W (-)	EBC Anal. (-)	EBC Calc. (-)	(Calc.-Anal.)/Anal. (%)
[illegible]	0.00000	0.00000	-
0.0016	1.20650	1.23155	2.0762[illegible]
[illegible]	[illegible]	[illegible]	1.8293[illegible]
[illegible]	2.77280	2.81592	1.5551[illegible]
[illegible]	3.09140	3.12444	1.04890
[illegible]	4.10890	4.14078	0.7759[illegible]
[illegible]	4.97540	4.96907	-0.1271[illegible]
[illegible]	6.78050	6.68616	-1.3913[illegible]
[illegible]	8.29330	8.17714	-1.4[illegible]065
[illegible]	11.41900	11.33678	-0.7024[illegible]
[illegible]	14.24500	14.33548	0.63520
[illegible]	17.24000	17.48594	1.4265[illegible]
[illegible]	19.66300	20.18365	2.64786
[illegible]	[illegible]	37.50277	1.3862[illegible]
[illegible]	[illegible]	53.96223	-1.4558[illegible]

RMSD ERROR = 0.3131

a/W = 0.60

ρ=r/W (-)	EBC Anal. (-)	EBC Calc. (-)	(Calc.-Anal.)/Anal. (%)
[illegible]	[illegible]	0.00000	-
[illegible]	[illegible]	1.90608	[illegible]
[illegible]	[illegible]	3.81724	[illegible]
[illegible]	[illegible]	4.35251	[illegible]
[illegible]	[illegible]	4.82966	[illegible]
[illegible]	[illegible]	6.40799	[illegible]
[illegible]	[illegible]	7.10674	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	13.32911	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]

RMSD ERROR = 0.7538

TABLE 12 (Concluded)

RESULTS FROM THE LINEAR EXPRESSION
PLUS CORRECTION TERM, EQ. (28), TO CALCULATE EBC

a/W = 0.70

ρ=r/W (-)	EBC Anal. (-)	EBC Calc. (-)	(Calc.-Anal.)/Anal. (%)
[illegible]	[illegible]	[illegible]	-
[illegible]	3.22100	3.26008	[illegible]
[illegible]	[illegible]	6.53060	[illegible]
[illegible]	[illegible]	7.45086	[illegible]
[illegible]	[illegible]	8.27443	[illegible]
[illegible]	[illegible]	11.03030	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]

RMSD ERROR = 1.3005

a/W = 0.80

ρ=r/W (-)	EBC Anal. (-)	EBC Calc. (-)	(Calc.-Anal.)/Anal. (%)
[illegible]	[illegible]	[illegible]	-
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]

RMSD ERROR = 3.2479

SECTION 5

APPLICATION

Two equations have been developed for compliance as a function of crack length for a CT specimen. For indirect crack length determination, it is desired to have crack length expressed in terms of compliance since compliance is measured in the experiment. A direct mathematical inversion is not easily obtained because both expressions are complicated functions of α. A curve fit of crack length as a function of both compliance and measurement location could be tedious and difficult. Thus, a numerical method, namely Newton's Method, is applied to Equation (28) to calculate values of crack length when compliance is known.

The problem is to find α when a compliance value C* is measured at a known location d, see Figure 2. From the form of (28), one can write

$$C^* = C(\alpha, \alpha - \delta),$$

where $\omega = \alpha - \delta$ and $\delta = d/W$, or

$$C^* = \hat{C}(\alpha), \tag{29}$$

where $\hat{C}$ represents a function of only crack length. An approximate solution for α in (29) can be found using Newton's Method. An iterative solution of (29) becomes

$$\alpha_i = \alpha_{i-1} - \frac{C^* - \hat{C}(\alpha_{i-1})}{\frac{d}{d\alpha}[\hat{C}]_{\alpha=\alpha_{i-1}}}, \ i=1,\ 2,\ \ldots \quad . \tag{30}$$

Since $\hat{C}(\alpha)$ is a complicated expression, the derivative was estimated in the numerical calculations as follows

$$\frac{d}{d\alpha}[\hat{C}] \approx \frac{\hat{C}(\alpha + \Delta/2) - \hat{C}(\alpha - \Delta/2)}{\Delta}, \tag{31}$$

where $\Delta \approx 0.05\ \alpha$.

To find α from (30) when (31) is substituted and the compliance has been determined, an initial estimate, $\alpha = \alpha_0$, is needed. Then, the iteration is terminated when $|\alpha_i - \alpha_{i-1}|$ is sufficiently small.

In the beginning of a crack growth test, α_0 could be determined from optical measurements. As the crack grows, a value of the initial crack length estimate for a subsequent compliance value can be the previously determined crack length.

The results of the iterative procedure, using (30) with the approximation for the derivative given by (31), are shown in Table 13. For these examples, values of compliance were chosen from the analytical values for $\alpha = 0.8$ used to obtain the compliance expression. In all examples, the initial value for α_0 was 0.7. Note that the number of iterations for convergence to a solution α^* is relatively small. The numerical iteration procedure was terminated when $|\alpha_i - \alpha_{i-1}| < 0.0001$.

TABLE 13

ITERATIVE SOLUTION FOR α USING EQ. (28)
WHEN C = C*

C^*	δ	α_o	α^*	Number of Iterations
304.60	0	0.70	0.798	8
118.87	0.5164	0.70	0.798	9
13.408	0.7900	0.70	0.799	8

SECTION 6

SUMMARY

Two empirical equations were developed for the displacement between the crack surfaces of a CT specimen. The equations were derived from curve fits to finite element results which were determined from a linear elastic analysis. Since a linear analysis was used, the displacements for any load were presented using compliance, the ratio of the change in displacement to the change in load. To avoid extensive curve fitting and to maintain some simplicity, the number of unknown coefficients used to fit the data for a given crack length was limited to two. Both equations yielded the correct analytical behavior as the crack tip was approached.

One equation was developed from a truncated series expansion for the square of the compliance in terms of integer powers of the square root of the distance from the crack tip. The unknown coefficients in the equation were determined from a least square error fit to the finite element results. The percent error between the calculated and the finite element results for compliance was within ± 3% over the ranges of $0 \leq \rho \leq a/W - 0.175$ and $0.25 \leq a/W \leq 0.8$. Since the truncated series was determined from curve fits to data within the above ranges, extrapolation outside of the ranges is not recommended.

The second equation was developed by adding a correction term to an approximate solution for the compliance far from the crack tip. The resulting expression was nonlinear in terms of the unknown coefficients. An iterative

trial-and-error method was combined with a least square error method to obtain solutions for the unknown coefficients. The percent error between the calculated and the finite element results was within ± 4% over the ranges $0 \leq \rho \leq a/W + 0.25$ and $0.25 \leq a/W \leq 0.8$. Note that the range of the distance from the crack tip is larger than the range for the truncated series.

Since the correction term was developed to account primarily for compliance in the vicinity of the crack tip, the second equation could be applied when values of ρ are beyond the indicated upper limit, $a/W + 0.25$.

In an application where compliance is measured and crack length is to be determined, a simple numerical iterative procedure, Newton's method, can be applied to Equation (28). A value for crack length can be readily determined while the number of iterations is somewhat insensitive to the initial estimate of crack length.

REFERENCES

1. Sharpe, W. N., Jr., "Interferometric Surface Strain Measurement," Int. J. of Nondestructive Testing, 3, 1971, pp. 59-76.

2. Williams, M. L., "On the Stress Distribution at the Base of a Stationary Crack," J. Appl. Mech., 25, 1957, pp. 109-114.

3. Orange, T. W., "Crack Shapes and Stress Intensity Factors for Edge-Cracked Specimens," Stress Analysis and Growth of Cracks, Proceedings of the 1971 National Symposium on Fracture Mechanics, Part I, ASTM STP 513, American Society for Testing and Materials, 1972, pp. 71-78.

4. Ahmad, J., "Two Dimensional Linear Elastic Analysis of Fracture Specimens - User's Manual of a Finite Element Computer Program," Technical Report AFWAL-TR-80-4008, Materials Laboratory, Air Force Wright Aeronautical Laboratories, Wright-Patterson Air Force Base, Ohio 45433, February 1980.

5. Srawley, J. E., "Wide Range Stress Intensity Factor Expressions for ASTM Method E399 Standard Fracture Toughness Specimens," International Journal of Fracture, 12, 1976, pp. 475-476.

6. Saxena, A. and Hudak, S. J., Jr., "Review and Extension of Compliance Information for Common Crack Growth Specimens," Int. Journal of Fracture, 14, pp. 453-468.

7. Newman, J. C., "Crack Opening Displacements in Center Crack, Compact and Crack Line Wedge Loaded Specimens," NASA TN-D-8268, Washington D.C., July 1976.

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